IGA/AMSI Workshop on

Group-valued moment maps with applications to mathematics and physics

The University of Adelaide, September 5–9, 2011

Lectures of Eckhard Meinrenken:

- *Introduction to G-valued moment maps*

  The theory of quasi-Hamiltonian G-spaces with G-valued moment maps has its origins in 2-dimensional gauge theory. Its features are similar to the usual Hamiltonian theory, but with interesting modifications. We will give an overview of the theory, with discussions of a convexity theorem and a Kirwan surjectivity theorem.

- *Dirac Geometry and Witten's volume formulas*

  Dirac geometry treats 2-forms and bivector fields within a common framework. We will explain how this leads to a conceptual approach to G-valued moment maps. As an application, we will construct Liouville volume forms, leading to a proof of Witten's volume formulas for moduli spaces of flat G-bundles over surfaces.

- *Dixmier–Douady theory and pre-quantization*

  Dixmier–Douady bundles provide geometric realizations of integral degree three cohomology classes over a space. We will use these bundles to construct distinguished "twisted spin^c structures" on quasi-Hamiltonian G-spaces, and also define pre-quantizations in similar terms.

- *Quantization of group-valued moment maps*

  We will review Rosenberg's definition of twisted K-homology in terms of Dixmier–Douady bundles and the Freed–Hopkins–Teleman theorem on the twisted K-homology of Lie groups. We then define the quantization of group-valued moment maps as push-forwards in twisted K-homology.

- *Application to Verlinde formulas*

  The quantization of a quasi-Hamiltonian space is computable via localization. In conjunction with a 'quantization commutes with reduction' theorem, this leads to the symplectic version of the Verlinde formulas for moduli spaces of flat G-bundles.

Other invited speakers:

1. **David Baraglia**, Australian National University

   Title: Monodromy and orientifolds in T-duality via Courant algebroids

   Abstract: T-duality in its simplest form is a duality between principal torus bundles equipped with flux (a gerbe). Here we consider two extensions to T-duality: torus bundles with non-trivial monodromy and spaces with involutions, a simple type of orientifold. We will show how using a Courant algebroid point of view leads naturally towards these extensions. T-duality entails an isomorphism in twisted K-theory. We show that in the presence of monodromy the isomorphism persists as long as we allow for twists by graded bundle gerbes. In the orientifold case we conjecture an isomorphism in twisted KR-theory and give some evidence to support this.

2. **Keith Hannabuss**, University of Oxford

   Title: Metaplectic moments

   Abstract: There are at least two exponentiated versions of the moment map: the group-valued moment map and the tempered distributions representing metaplectic automorphisms of the Moyal algebra. Although defined in totally different contexts there is a common domain in which both of these make sense. In this talk we shall explain the background to the Moyal algebra and metaplectic representation, and show the simple connection with the group-valued moment map, which also extends to exponential automorphisms of star product algebras on homogeneous spamplectic spaces.

3. **John Huerta**, Australian National University

   Title: A higher supergroup for string theory

   Abstract: Classically, superstring theory makes sense in spacetimes of dimension 3, 4, 6 and 10. It is no coincidence that these numbers are two more than 1, 2, 4 and 8, the dimension of the normed division algebras. I will describe how this known relationship leads to "higher gauge theory". Just as gauge theory describes the parallel transport of supersymmetric particles using Lie supergroups, higher gauge theory describes the parallel transport of superstrings using "Lie 2-supergroups". With the aid of the normed division algebras and some elementary geometry, one can construct a Lie 2-supergroup extending the Poincaré supergroup when spacetime has dimension 3, 4, 6 and 10.

4. **Paul Sobaje**, University of Melbourne

   Title: Support varieties and other module invariants

   Abstract: Over the last few decades, cohomological support varieties for modules have become an increasingly useful tool within representation theory, and have been applied in studying the representations of many different objects, including: finite groups, Frobenius kernels of algebraic groups, and small quantum groups. In this talk we will start with a

general introduction to this theory, and then look at some recent developments, including a method due to E. Friedlander and J. Pevtsova which uses this theory to produce algebraic vector bundles on projective space.

5. **Stephan Tillmann**, University of Queensland

   Title: Volume optimisation on triangulated 3-manifolds

   Abstract: In 1978, Thurston introduced an affine algebraic set to study hyperbolic structures on triangulated 3-manifolds. Recently, Feng Luo discovered a finite-dimensional variational principle on triangulated 3-manifolds with the property that its critical points are related to both Thurston's algebraic set and to Haken's normal surface theory. The action functional is the volume. This is a generalisation of an earlier program by Casson and Rivin for compact 3-manifolds with torus boundary.

   In this talk, I will describe algorithms for 3-manifolds using volume maximisation. Part of this talk is based on joint work with Feng Luo and Tian Yang.

6. **Craig Westerland**, University of Melbourne

   Title: *Twisted Morava K-theory*

   Abstract: Morava's extraordinary K-theories $K(n)$ are a family of generalized cohomology theories which behave in some ways like K-theory (indeed, $K(1)$ is mod 2 K-theory). Their construction exploits Quillen's description of cobordism in terms of formal group laws and Lubin–Tate's methods in class field theory for constructing abelian extensions of number fields. Constructed from homotopy-theoretic methods, they do not admit a geometric description (like deRham cohomology, K-theory, or cobordism), but are nonetheless subtle, computable invariants of topological spaces. In this talk, I will give an introduction to these theories, and explain how it is possible to define an analogue of twisted K-theory in this setting. Traditionally, K-theory is twisted by a three-dimensional cohomology class; in this case, $K(n)$ admits twists by $(n+2)$-dimensional classes. This work is joint with Hisham Sati.

7. **Ruibin Zhang**, University of Sydney

   Title: Equivariant algebraic K-theory of quantum group actions

   Abstract: Let U_q be a quantum group. Regarding a noncommutative space with U_q-symmetry as a U_q-module algebra A, we may think of equivariant vector bundles on A as projective A-modules with compatible U_q-action. We construct an equivariant K-theory of such quantum vector bundles using Quillen's exact categories, and provide means for its computation. The equivariant K-groups of quantum homogeneous spaces and quantum symmetric algebras of classical type are computed. This is joint work with Gus Lehrer.