Dirac Operators in Geometry, Topology, Representation Theory and Physics

Conference Room 7.15, Level 7, Innova 21 building

School of Mathematical Sciences, The University of Adelaide

October 18–22, 2010

IGA/AMSI Workshop

Abstracts of talks

Lectures of Dan Freed, University of Texas at Austin

- *Loop groups and Dirac families*

  I will describe the finite and infinite-dimensional versions of the construction which yields my theorem with Hopkins and Teleman. This will include some generalities about loop groups and their representations.

- *Introduction to twisted K-theory*

  After a review of standard K-theory I will discuss the twisted form and some properties. Then I will compute twisted $K_G(G)$ in some cases.

- *A TQFT from twisted K-theory*

  I will discuss TQFTs in general and give some examples, etc. I will show how to construct the ring structure on twisted $K_G(G)$.

- *Differential K-theory and the Atiyah-Singer theorem*

  An introductory lecture on differential cohomology theories in general and differential K-theory in particular. Then the differential index theorem with Lott.

- *Anomalies and "categorified" index theorems*

  A general lecture on some physics, then some discussion of anomalies in supersymmetric quantum mechanics and on the worldsheet of string theory.

Other invited speakers:

1. **Michael Batanin**, Macquarie University

   Title: Kan extension and classification theorems

   Abstract: The concept of Kan extension is one of the most basics concepts in category theory. Given a functor between categories $f : C \to D$ one can construct a restriction functor res_f along f between functor categories $f : [D, V] \to [C, V]$. The left adjoint (if exists) to res_f is called left Kan extension and the right adjoint is called right Kan extension along f. It is not difficult to see that this definition is purely formal and can be modified to make sense in any 2-category. There is also a derived version of this concept called the homotopy Kan extension. There are powerful tools in category theory and homotopy theory for computing (derived) Kan extensions.

   In this talk I will show that Kan extension is also fundamental in many areas outside category theory. Many important classification results like recognition principle for n-fold loop spaces, Breen-Baez-Dolan stabilisation hypothesis and classification theorems for various types of field theories can be obtained by computing an appropriate (derived) left Kan extension.

2. **Arun Ram**, University of Melbourne

   Title: Elliptic cohomology and Weyl character formulas

   Abstract: In this work, joint with Nora Ganter, we establish an elliptic cohomology version of the Atiyah–Segal–Lefschetz fixed point formula and apply it to the flag variety of a compact Lie group. We make contact with the work of Looijenga on Root Systems and Elliptic Curves and the work of Kac and Peterson on Affine Lie algebras and Modular Forms and obtain Weyl characters for the loop group as push forwards in elliptic cohomology.

3. **David Ridout**, Australian National University

   Title: D-Brane Charges in Wess–Zumino–Witten Models

   Abstract: The phenomenon of D-brane condensation, specifically the charge which is conserved by this process, is well-known to be historically linked to the computation of twisted K-theory groups. In this talk, I will review this story before discussing a less popular, though more geometric, approach to brane charges. We will see that the celebrated results of the condensation approach are completely reproduced by recognising various consistency requirements. In other words, the dynamical constraints on the charges are in fact already accounted for in the geometry.

4. **Frederic Rochon**, Australian National University

   Title: Dirac operators on manifolds with foliated boundaries

   Abstract: After describing the pseudodifferential calculus that Mazzeo and Melrose introduced on a manifold with fibred boundary, we will indicate how to generalise their construction to situations where the fibration on the boundary is replaced by a foliation. The operators obtained in this way have nice mapping properties. We will provide some simple criteria to determine when such operators are compact or Fredholm (when acting on suitable Sobolev spaces). We will conclude by exhibiting a formula for the index of certain Dirac-type operators arising in this context.

5. **Anne Thomas**, University of Sydney

   Title: Lattices in complete Kac-Moody groups

   Abstract: A complete Kac-Moody group over a finite field is a totally disconnected, locally compact group, which may be thought of as an "infinite-dimensional Lie group". An example is $G = SL(n, K)$ with K the field of formal Laurent series over a finite field. We study uniform and nonuniform lattices in such G where the associated Bruhat-Tits building is a tree. We use finite group theory and the dynamics of the group action on the tree and its boundary. This is joint work with Inna (Korchagina) Capdeboscq.

6. **Craig Westerland**, University of Melbourne

   Title: $\mathbb{T}$-duality and Atiyah duality

   Abstract: Bouwknegt, Evslin, Mathai and a host of other authors have investigated the physics and topology of $\mathbb{T}$-duality. One consequence of this duality is an isomorphism of the twisted K-theories of $\mathbb{T}$-dual circle bundles. We reinterpret this result in the language of stable homotopy theory, realizing it through an exotic nondegenerate pairing of modules over the K-theory spectrum. Additionally, we give a new proof (based on [BEM]) in the language of equivariant homotopy theory.