AMSI/IGA workshop on

Representation theory and operator algebras

University of Adelaide, 1–5 July 2013

Lecture series by Nigel Higson

Geometry, Noncommutative Geometry and Group Representations

Abstract:
These talks will be about aspects of representation theory (in roughly the sense of Harish-Chandra) that can be illuminated using ideas from noncommutative geometry (in roughly the sense of Alain Connes, in which operator algebras are used to help express geometric concepts). I'll start with Weyl's famous character formula for compact Lie groups. I'll review his derivation of the formula, his simultaneous classification of the representations of a compact Lie group, and the subsequent reinterpretation of Weyl's formula by Atiyah and Bott using fixed-point theory. From there I'll move towards noncompact groups. I'll sketch some of the work of Harish-Chandra, and then describe several noncommutative-geometric perspectives on the representation theory of noncompact reductive groups.

1. The Geometry of the Weyl Character Formula
2. A Sketch of Harish-Chandra's Theory for Noncompact Groups
3. Baum-Connes versus Beilinson-Bernstein
4. Contractions of Lie Groups and the Mackey Analogy
5. On the Category of Tempered Representations

Talks by invited speakers

Michael Cowling

Uniformly bounded representations and completely bounded multipliers

Abstract:
Every locally compact group G has a Fourier algebra $A(G)$, the space of matrix coefficients of the (left) regular representation of G on $L^2(G)$, which is a Banach algebra under pointwise operations; its dual is the von Neumann algebra $VN(G)$ of linear operators on $L^2(G)$ that commute with (right) translations; these may be thought of as convolutions with distributions. There is some interest in the (pointwise) multipliers of $A(G)$, and particularly in the completely bounded multipliers of $A(G)$, those whose transpose is a completely bounded map of $VN(G)$. Questions of interest include whether the constant function 1 may be approximated, locally uniformly say, by completely bounded multipliers of $A(G)$ that vanish at infinity, and whether this can be done keeping control of the completely bounded multiplier norm of the approximants. One of the difficulties is that it is hard to compute completely bounded norms.

All matrix coefficients of unitary, and more generally uniformly bounded, representations of G are completely bounded multipliers of $A(G)$. This talk is a survey of uniformly bounded representations and completely bounded multipliers on semisimple Lie groups.

Loek Helminck

Orbits and invariants associated with generalized symmetric spaces

Abstract:
Symmetric spaces occur in many areas of mathematics and physics, probably best known are their applications in Lie theory, differential geometry and harmonic analysis. In this talk we will discuss a generalization of these symmetric spaces, which has become of importance in many areas of mathematics. In particular we will consider orbits of parabolic and symmetric subgroups acting on these generalized symmetric spaces, which play a fundamental role in the study of representations associated with these generalized symmetric spaces.

Peter Hochs

Quantisation and K-theory

Abstract:
In geometric quantisation, one constructs unitary representations from symplectic manifolds, as equivariant indices of Dirac operators. For compact groups acting on compact symplectic manifolds, those indices are well-defined, finite-dimensional virtual representations. For noncompact groups and/or manifolds, one needs to use more general indices to define geometric quantisation. I will discuss an approach to the *quantisation commmutes with reduction* problem in the noncompact setting. In this approach, quantisation is defined in terms of index maps with values in the K-theory groups of certain C^*-algebras. In many

cases, these K-theory groups contain classes associated to irreducible representations of the group in question. The talk includes joint work with Mathai Varghese.

Tony Licata

Representation theory from the McKay correspondence

Abstract:
Conjugacy classes of finite nontrivial subgroups of SU(2) are in 1-1 correspondence with simply-laced affine Dynkin diagrams; this particular ADE classification is referred to as the McKay correspondence. In principle, then, one should be able to start with a finite subgroup of SU(2) and use it to produce representations of the corresponding affine Lie algebra. The goal of this talk will be to give a modern version of such a construction, using ideas from categorification and geometric representation theory.

Adam Rennie

Circle equivariant KK-theory and KMS states

Abstract:
I will describe the way circle equivariant KK-theory provides an interpretation of the 'modular' index theory of KMS states. This generalises to an interpretation for some higher dimensional examples, but much remains to be understood.

This is joint work with A. Carey, S. Neshveyev, R. Nest, U. Kraehmer, R. Senior.

Aidan Sims

Simplicity and Morita equivalence for C*-algebras of étale groupoids

Abstract:
An étale groupoid is a discrete group with an identity crisis: it has multiple identities, no two of which can sensibly be multiplied. I will describe the convolution algebra of an étale groupoid, and describe a recent characterisation of when the universal C^*-completion of this convolution algebra is simple. I will then describe Renault's notion of equivalence of groupoids, and discuss a groupoid analogue of Green's imprimitivity theorem for group crossed-products for both the full and reduced C^*-algebra.

This talk includes results from joint work with Jon Brown, Lisa Orloff Clark, Cindy Farthing and Dana Williams.