Higher Twisted K-theory

Analysis on Manifolds

David Brook

Supervisors: Elder Professor Mathai Varghese and Dr Peter Hochs
The University of Adelaide

1.10.2019

Topological K-theory

Topological K-theory is a generalised cohomology theory.

Topological K-theory

Topological K-theory is a generalised cohomology theory.

- For compact Hausdorff X, define $K^n(X) = [X, \Omega^n \operatorname{Fred}]$;

Topological K-theory

Topological K-theory is a generalised cohomology theory.

- For compact Hausdorff X, define $K^n(X) = [X, \Omega^n \mathrm{Fred}]$;
- Bott periodicity: $K^n(X) \cong K^{n+2}(X)$;

Topological K-theory

Topological K-theory is a generalised cohomology theory.

- For compact Hausdorff X, define $K^n(X) = [X, \Omega^n \mathrm{Fred}]$;
- Bott periodicity: $K^n(X) \cong K^{n+2}(X)$;
- There is a notion of algebraic K-theory K_* such that $K^*(X) = K_*(C(X))$.

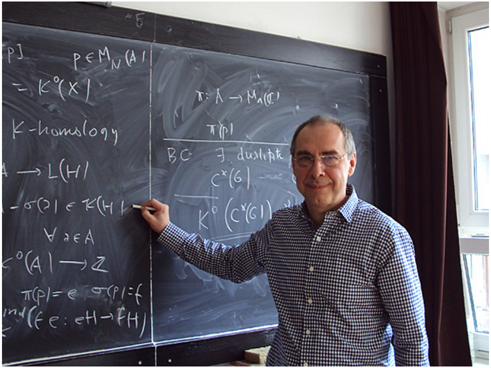
Topological K-theory

Topological K-theory is a generalised cohomology theory.

- For compact Hausdorff X, define $K^n(X) = [X, \Omega^n \text{Fred}]$;
- Bott periodicity: $K^n(X) \cong K^{n+2}(X)$;
- There is a notion of algebraic K-theory K_* such that $K^*(X) = K_*(C(X))$.

As with all generalised cohomology theories, there exists a notion of "twist" for K-theory.

Pennig and Dadarlat



The **Cuntz algebra** $\mathcal{O}_\infty$ with infinitely many generators is defined to be the C^*-algebra generated by a set of isometries $\{S_i\}_{i\in\mathbb{N}}$ acting on a separable Hilbert space satisfying

$$\sum_{i=1}^{k} S_i S_i^* \leq I$$

for all $k \in \mathbb{N}$.

The Cuntz algebra

The **Cuntz algebra** $\mathcal{O}_\infty$ with infinitely many generators is defined to be the C^*-algebra generated by a set of isometries $\{S_i\}_{i\in\mathbb{N}}$ acting on a separable Hilbert space satisfying

$$\sum_{i=1}^{k} S_i S_i^* \leq I$$

for all $k \in \mathbb{N}$.

Pennig and Dadarlat show that $\mathcal{O}_\infty \otimes \mathcal{K}$ provides a geometric interpretation of the higher twists: twists of $K^*(X)$ can be identified with algebra bundles over X with fibre $\mathcal{O}_\infty \otimes \mathcal{K}$.

Physical motivation

Let M be a spacetime, and $\mathcal{O}$ an algebra bundle over M with fibre $\mathcal{O}_\infty \otimes \mathcal{K}$.

Physical motivation

Let M be a spacetime, and $\mathcal{O}$ an algebra bundle over M with fibre $\mathcal{O}_\infty \otimes \mathcal{K}$. If $\{U_i\}_{i\in I}$ is a trivialising open cover for $\mathcal{O}$ consisting of contractible sets, then locally the K-theory of $\mathcal{O}$ is given by

$$\begin{aligned} K^n(U_i \times (\mathcal{O}_\infty \otimes \mathcal{K})) &= (K^0(U_i) \otimes K_n(\mathcal{O}_\infty \otimes \mathcal{K})) \\ &\quad \oplus (K^1(U_i) \otimes K_{n+1}(\mathcal{O}_\infty \otimes \mathcal{K})) \\ &= (K^0(U_i) \otimes K_n(\mathcal{O}_\infty)) \\ &\quad \oplus (K^1(U_i) \otimes K_{n+1}(\mathcal{O}_\infty)) \\ &= K^n(U_i). \end{aligned}$$

Physical motivation

Let M be a spacetime, and $\mathcal{O}$ an algebra bundle over M with fibre $\mathcal{O}_\infty \otimes \mathcal{K}$. If $\{U_i\}_{i\in I}$ is a trivialising open cover for $\mathcal{O}$ consisting of contractible sets, then locally the K-theory of $\mathcal{O}$ is given by

$$\begin{aligned} K^n(U_i \times (\mathcal{O}_\infty \otimes \mathcal{K})) &= (K^0(U_i) \otimes K_n(\mathcal{O}_\infty \otimes \mathcal{K})) \\ &\quad \oplus (K^1(U_i) \otimes K_{n+1}(\mathcal{O}_\infty \otimes \mathcal{K})) \\ &= (K^0(U_i) \otimes K_n(\mathcal{O}_\infty)) \\ &\quad \oplus (K^1(U_i) \otimes K_{n+1}(\mathcal{O}_\infty)) \\ &= K^n(U_i). \end{aligned}$$

Locally, the K-theory of $\mathcal{O}$ is given by that of the spacetime, while globally they are different.

Higher twisted K-theory

Definition

Let X be a compact Hausdorff space and $\mathcal{E}_\delta \to X$ an algebra bundle with fibre $\mathcal{O}_\infty \otimes \mathcal{K}$ representing a twist δ of $K^*(X)$. The K-theory of X twisted by δ is $K^n(X;\delta) := K_n(C(X, \mathcal{E}_\delta))$.

A higher Dixmier-Douady invariant

Theorem (Pennig-Dadarlat 2016)

Let X be a finite connected CW complex such that $H^*(X, \mathbb{Z})$ is torsion-free. Then

$$\mathrm{Bun}_{\mathcal{O}_\infty \otimes \mathcal{K}}(X) \cong H^1(X, \mathbb{Z}_2) \oplus \bigoplus_{k \geq 1} H^{2k+1}(X, \mathbb{Z}).$$

A higher Dixmier-Douady invariant

Theorem (Pennig-Dadarlat 2016)

Let X be a finite connected CW complex such that $H^*(X,\mathbb{Z})$ is torsion-free. Then

$$\mathrm{Bun}_{\mathcal{O}_\infty \otimes \mathcal{K}}(X) \cong H^1(X,\mathbb{Z}_2) \oplus \bigoplus_{k \geq 1} H^{2k+1}(X,\mathbb{Z}).$$

Define higher Dixmier-Douady invariants

$$\delta_k : \mathrm{Bun}_{\mathcal{O}_\infty \otimes \mathcal{K}}(X) \to H^{2k+1}(X,\mathbb{Z})$$

using this result.

Results for SU(n)

Theorem

For any non-zero $h_5 \in H^5(\mathrm{SU}(n+1), \mathbb{Z})$ *relatively prime to* $n!$ $(n > 1)$, $K^*(\mathrm{SU}(n+1), h_5)$ *is isomorphic to* $\mathbb{Z}_{|h_5|}$ *tensored with an exterior algebra on* $n-1$ *odd generators.*

Results for SU(n)

Theorem

For any non-zero $h_5 \in H^5(\mathrm{SU}(n+1), \mathbb{Z})$ *relatively prime to* $n!$ ($n > 1$), $K^*(\mathrm{SU}(n+1), h_5)$ *is isomorphic to* $\mathbb{Z}_{|h_5|}$ *tensored with an exterior algebra on* $n-1$ *odd generators.*
More generally, if $h \in H^{odd}(\mathrm{SU}(n), \mathbb{Z})$ *is non-zero then* $K^*(\mathrm{SU}(n), h)$ *is a finite abelian group and all elements have order a divisor of a power of* $|h|$.

Results for SU(n)

Theorem

For any non-zero $h_5 \in H^5(\mathrm{SU}(n+1), \mathbb{Z})$ *relatively prime to* $n!$ *(*$n > 1$*),* $K^*(\mathrm{SU}(n+1), h_5)$ *is isomorphic to* $\mathbb{Z}_{|h_5|}$ *tensored with an exterior algebra on* $n-1$ *odd generators.*
More generally, if $h \in H^{odd}(\mathrm{SU}(n), \mathbb{Z})$ *is non-zero then* $K^*(\mathrm{SU}(n), h)$ *is a finite abelian group and all elements have order a divisor of a power of* $|h|$*.*

Outline of proof

The first statement is proved via induction on n. The base case $n = 2$ can be proved using the Atiyah-Hirzebruch spectral sequence, and the inductive step follows from the Segal spectral sequence.

Results for SU(n)

Theorem

For any non-zero $h_5 \in H^5(\mathrm{SU}(n+1), \mathbb{Z})$ relatively prime to $n!$ ($n > 1$), $K^*(\mathrm{SU}(n+1), h_5)$ is isomorphic to $\mathbb{Z}_{|h_5|}$ tensored with an exterior algebra on $n-1$ odd generators.
More generally, if $h \in H^{odd}(\mathrm{SU}(n), \mathbb{Z})$ is non-zero then $K^*(\mathrm{SU}(n), h)$ is a finite abelian group and all elements have order a divisor of a power of $|h|$.

Outline of proof

The first statement is proved via induction on n. The base case $n = 2$ can be proved using the Atiyah-Hirzebruch spectral sequence, and the inductive step follows from the Segal spectral sequence.
The second statement also follows from the Segal spectral sequence.

Atiyah-Hirzebruch spectral sequence

Theorem

Let X be a finite CW complex with torsion-free cohomology and $h \in H^{2n+1}(X, \mathbb{Z})$. There is a strongly convergent Atiyah-Hirzebruch spectral sequence converging to $K^*(X, h)$ with E_2-term

$$E_2^{p,q} = H^p(X, K^q(pt)).$$

The differential $d_{2n+1} : H^j(X, \mathbb{Z}) \to H^{j+2n+1}(X, \mathbb{Z})$ is given by a twisted Steenrod operation: $d_{2n+1}(x) = Sq^{2n+1}(x) + x \cup h$.

Atiyah-Hirzebruch spectral sequence

The existence of the spectral sequence is established via the skeletal filtration of X, using the p-skeleton X^p to define the filtration

$$K_p^n(X) = \ker[K^n(X,h) \to K_n(\mathcal{E}_h|_{X^{p-1}})]$$

of $K^n(X,h)$.

Atiyah-Hirzebruch spectral sequence

The existence of the spectral sequence is established via the skeletal filtration of X, using the p-skeleton X^p to define the filtration

$$K_p^n(X) = \ker[K^n(X, h) \to K_n(\mathcal{E}_h|_{X^{p-1}})]$$

of $K^n(X, h)$. Standard arguments show that the E_2 term is of the form $E_2^{p,q} = H^p(X, K^q(pt))$.

Atiyah-Hirzebruch spectral sequence

The existence of the spectral sequence is established via the skeletal filtration of X, using the p-skeleton X^p to define the filtration

$$K_p^n(X) = \ker[K^n(X,h) \to K_n(\mathcal{E}_h|_{X^{p-1}})]$$

of $K^n(X,h)$. Standard arguments show that the E_2 term is of the form $E_2^{p,q} = H^p(X, K^q(pt))$.

The d_{2n+1} differential must be a universal cohomology operation raising degree by $2n+1$ defined for spaces with a given $h \in H^{2n+1}(X,\mathbb{Z})$. Standard arguments in homotopy theory show that these operations are classified by

$$H^{p+2n+1}(K(\mathbb{Z},p) \times K(\mathbb{Z},2n+1), \mathbb{Z}),$$

from which we conclude that $d_{2n+1}(x) = Sq^{2n+1}(x) + x \cup h$.

Segal spectral sequence

Theorem

Let $F \xrightarrow{\iota} E \xrightarrow{\pi} B$ be a fibre bundle of CW complexes, and let $h \in H^{odd}(E, \mathbb{Z})$. Then there is a homological spectral sequence

$$H_p(B, K_q(F, \iota^* h)) \Rightarrow K_*(E, h)$$

and a corresponding cohomological spectral sequence

$$H^p(B, K^q(F, \iota^* h)) \Rightarrow K^*(E, h).$$

These spectral sequences are strongly convergent if the ordinary (co)homology of B is bounded.

Differentials in Segal spectral sequence

Theorem

In the setting of the homology Segal spectral sequence, suppose that

Differentials in Segal spectral sequence

Theorem

In the setting of the homology Segal spectral sequence, suppose that

- ι^* *is an isomorphism on* H^{2n+1} *so that the twisting class on* E *can be identified with the restricted twisting class on* F,

Differentials in Segal spectral sequence

Theorem

In the setting of the homology Segal spectral sequence, suppose that

- ι^* *is an isomorphism on* H^{2n+1} *so that the twisting class on* E *can be identified with the restricted twisting class on* F*,*
- *the differentials* $d^2, \cdots, d^{r-1}$ *leave* $E^2_{r,0} = H_r(B, K_0(F, \iota^* h))$ *unchanged, and*

Differentials in Segal spectral sequence

Theorem

In the setting of the homology Segal spectral sequence, suppose that

- ι^* *is an isomorphism on* H^{2n+1} *so that the twisting class on* E *can be identified with the restricted twisting class on* F,
- *the differentials* $d^2, \cdots, d^{r-1}$ *leave* $E^2_{r,0} = H_r(B, K_0(F, \iota^* h))$ *unchanged, and*
- *there is a class* $x \in E^2_{r,0}$ *which comes from a class* $\alpha \in \pi_r(B)$ *under the Hurewicz map* $\pi_r(B) \to H_r(B, K_0(F, \iota^* h))$.

Differentials in Segal spectral sequence

Theorem

In the setting of the homology Segal spectral sequence, suppose that

- ι^* *is an isomorphism on* H^{2n+1} *so that the twisting class on* E *can be identified with the restricted twisting class on* F,
- *the differentials* $d^2, \cdots, d^{r-1}$ *leave* $E^2_{r,0} = H_r(B, K_0(F, \iota^* h))$ *unchanged, and*
- *there is a class* $x \in E^2_{r,0}$ *which comes from a class* $\alpha \in \pi_r(B)$ *under the Hurewicz map* $\pi_r(B) \to H_r(B, K_0(F, \iota^* h))$.

Then $d^r(x) \in E^r_{0,r-1}$ *is the image of* α *under the composition of the boundary map* $\partial : \pi_r(B) \to \pi_{r-1}(F)$ *in the long exact sequence of the fibration and the Hurewicz map* $\pi_{r-1}(F) \to K_{r-1}(F, \iota^* h)$.

Proof

Without loss of generality, take B to be S^r and $E = (\mathbb{R}^r \times F) \cup F$, where $\mathbb{R}^r \times F$ is π^{-1} of the open r-cell in B. Then the spectral sequence comes from the long exact sequence

$$\cdots \to K_r(F, \iota^* h) \xrightarrow{\iota_*} K_r(E, h) \to K_r(E, F, h) \\ \cong K_r(E \setminus F, h) \cong K_0(F, \iota^* h) \xrightarrow{\partial} K_{r-1}(F, \iota^* h) \to \cdots$$

where we identify $K_0(F, \iota^* h)$ with $H_r(B, K_0(F, \iota^* h))$.

Proof

Without loss of generality, take B to be S^r and $E = (\mathbb{R}^r \times F) \cup F$, where $\mathbb{R}^r \times F$ is π^{-1} of the open r-cell in B. Then the spectral sequence comes from the long exact sequence

$$\cdots \to K_r(F, \iota^* h) \xrightarrow{\iota_*} K_r(E, h) \to K_r(E, F, h)$$
$$\cong K_r(E \setminus F, h) \cong K_0(F, \iota^* h) \xrightarrow{\partial} K_{r-1}(F, \iota^* h) \to \cdots$$

where we identify $K_0(F, \iota^* h)$ with $H_r(B, K_0(F, \iota^* h))$. Hence the differential d^r is simply the boundary map in this sequence, and the result follows from the naturality of the Hurewicz homomorphism which implies the commutativity of the diagram

$$\begin{array}{ccc} \pi_r(B) & \xrightarrow{\quad\partial\quad} & \pi_{r-1}(F) \\ {\scriptstyle \text{Hurewicz}}\downarrow & & \downarrow{\scriptstyle \text{Hurewicz}} \\ H_r(B, K_0(F, \iota^* h)) & \xrightarrow{\partial} & K_{r-1}(F, \iota^* h). \end{array}$$

Higher twisted K-homology

Proposition

If the higher twisted K-homology of X is torsion, then the higher twisted K-theory and higher twisted K-homology of X are (non-canonically) isomorphic.

Higher twisted K-homology

Proposition

If the higher twisted K-homology of X is torsion, then the higher twisted K-theory and higher twisted K-homology of X are (non-canonically) isomorphic.

Proof

Higher twisted K-theory is the operator algebraic K-theory of a section algebra A, and higher twisted K-homology is the KK-theory $KK_*(A, \mathcal{O}_\infty)$. These groups are related by a special case of the universal coefficient theorem in KK-theory, which can be stated as

$$0 \to \mathrm{Ext}^1_{\mathbb{Z}}(K_{\bullet+1}(A), \mathbb{Z}) \to KK_\bullet(A, \mathcal{O}_\infty) \to \mathrm{Hom}_{\mathbb{Z}}(K_\bullet(A), \mathbb{Z}) \to 0.$$

Higher twisted K-homology

Proposition

If the higher twisted K-homology of X is torsion, then the higher twisted K-theory and higher twisted K-homology of X are (non-canonically) isomorphic.

Proof

Higher twisted K-theory is the operator algebraic K-theory of a section algebra A, and higher twisted K-homology is the KK-theory $KK_*(A, \mathcal{O}_\infty)$. These groups are related by a special case of the universal coefficient theorem in KK-theory, which can be stated as

$$0 \to \mathrm{Ext}^1_{\mathbb{Z}}(K_{\bullet+1}(A), \mathbb{Z}) \to KK_\bullet(A, \mathcal{O}_\infty) \to \mathrm{Hom}_{\mathbb{Z}}(K_\bullet(A), \mathbb{Z}) \to 0.$$

If $KK_\bullet(A, \mathcal{O}_\infty)$ is torsion then $K_\bullet(A)$ is also torsion, and hence the groups agree except for a degree shift.

Base case: 5-twisted K-theory of SU(3)

Let $h_5 \in H^5(\mathrm{SU}(3), \mathbb{Z})$.

2	$\mathbb{Z}$	0	0	$\mathbb{Z}c_3$	0	$\mathbb{Z}c_5$	0	0	$\mathbb{Z}c_3 \wedge \mathbb{Z}c_5$
1	0	0	0	0	0	0	0	0	0
0	$\mathbb{Z}$	0	0	$\mathbb{Z}c_3$	0	$\mathbb{Z}c_5$	0	0	$\mathbb{Z}c_3 \wedge \mathbb{Z}c_5$
	0	1	2	3	4	5	6	7	8

The only non-trivial differential is $d_5(x) = \mathrm{Sq}^5(x) + x \cup h_5$.

2	0	0	0	0	0	$\mathbb{Z}_{\|h_5\|}$	0	0	$\mathbb{Z}_{\|h_5\|}$
1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	$\mathbb{Z}_{\|h_5\|}$	0	0	$\mathbb{Z}_{\|h_5\|}$
	0	1	2	3	4	5	6	7	8

Hence $K^0(\mathrm{SU}(3), h_5) \cong \mathbb{Z}_{|h_5|}$ and $K^1(\mathrm{SU}(3), h_5) \cong \mathbb{Z}_{|h_5|}$. So $K^*(\mathrm{SU}(3), h_5)$ is of the form $\mathbb{Z}_{|h_5|}$ tensored with $\mathbb{Z}c$ for some odd generator c.

Inductive step

Assume that $n > 2$ and that the result holds for smaller values of n. The Segal spectral sequence associated to the fibration

$$\mathrm{SU}(n) \xrightarrow{\iota} \mathrm{SU}(n+1) \to S^{2n+1}$$

gives

$$E^2_{p,q} = H_p(S^{2n+1}, K_q(\mathrm{SU}(n), h_5)) \Rightarrow K_*(\mathrm{SU}(n+1), h_5).$$

Since h_5 is relatively prime to $(n-1)!$, by the inductive assumption $K_*(\mathrm{SU}(n), h_5) \cong \mathbb{Z}_{|h_5|} \otimes \wedge(x_1, \cdots, x_{n-2})$ for some odd generators x_i. We just need to show that the spectral sequence collapses.

Collapse of the spectral sequence

The only potentially non-zero differential is d^{2n+1}, which comes from the Hurewicz maps and the long exact sequence in homotopy for the fibration. This long exact sequence contains

$$\pi_{2n+1}(\mathrm{SU}(n+1)) \to \pi_{2n+1}(S^{2n+1}) \xrightarrow{\partial} \pi_{2n}(\mathrm{SU}(n)) \to \pi_{2n}(\mathrm{SU}(n+1)),$$

so we see that the boundary map $\partial : \mathbb{Z} \to \mathbb{Z}_{n!}$ has kernel of index $n!$.

Collapse of the spectral sequence

The only potentially non-zero differential is d^{2n+1}, which comes from the Hurewicz maps and the long exact sequence in homotopy for the fibration. This long exact sequence contains

$$\pi_{2n+1}(\mathrm{SU}(n+1)) \to \pi_{2n+1}(S^{2n+1}) \xrightarrow{\partial} \pi_{2n}(\mathrm{SU}(n)) \to \pi_{2n}(\mathrm{SU}(n+1)),$$

so we see that the boundary map $\partial : \mathbb{Z} \to \mathbb{Z}_{n!}$ has kernel of index $n!$. The Hurewicz map of interest is

$$\pi_{2n}(\mathrm{SU}(n)) \to K_{2n}(\mathrm{SU}(n), h_5) \cong K_0(\mathrm{SU}(n), h_5).$$

Collapse of the spectral sequence

The only potentially non-zero differential is d^{2n+1}, which comes from the Hurewicz maps and the long exact sequence in homotopy for the fibration. This long exact sequence contains

$$\pi_{2n+1}(\mathrm{SU}(n+1)) \to \pi_{2n+1}(S^{2n+1}) \xrightarrow{\partial} \pi_{2n}(\mathrm{SU}(n)) \to \pi_{2n}(\mathrm{SU}(n+1)),$$

so we see that the boundary map $\partial : \mathbb{Z} \to \mathbb{Z}_{n!}$ has kernel of index $n!$. The Hurewicz map of interest is

$$\pi_{2n}(\mathrm{SU}(n)) \to K_{2n}(\mathrm{SU}(n), h_5) \cong K_0(\mathrm{SU}(n), h_5).$$

Since this is a map $\mathbb{Z}_{n!} \to \mathbb{Z}_{|h_5|}$, if $\gcd(|h_5|, n!) = 1$ then this map must be trivial and hence the differential is trivial. Thus if $\gcd(|h_5|, n!) = 1$ then $K_*(\mathrm{SU}(n+1), h_5)$ is isomorphic to $\mathbb{Z}_{|h_5|}$ tensored with an exterior algebra on $n-1$ odd generators as required.

Proof of second statement

Generalising the computation of $K^*(\mathrm{SU}(3), h_5)$ shows that $K^i(\mathrm{SU}(n), h_{2n-1})$ is a torsion group whose elements have order a divisor of a power of $|h_{2n-1}|$.

Proof of second statement

Generalising the computation of $K^*(\mathrm{SU}(3), h_5)$ shows that $K^i(\mathrm{SU}(n), h_{2n-1})$ is a torsion group whose elements have order a divisor of a power of $|h_{2n-1}|$. Again applying the Segal spectral sequence to the fibration over S^{2n+1} in cohomology gives

$$E_2^{p,q} = H^p(S^{2n+1}, K^q(\mathrm{SU}(n), h_{2n-1})) \Rightarrow K^*(\mathrm{SU}(n+1), h_{2n-1}).$$

But $K^q(\mathrm{SU}(n), h_{2n-1})$ is torsion with all elements of order a divisor of a power of $|h_{2n-1}|$, and so the same is true for E_2 and thus E_∞. Finally, even if there are non-trivial extension problems to solve in order to obtain $K^*(\mathrm{SU}(n+1), h_{2n-1})$, the result is still true.

Other work

- Developed a product on higher twisted K-theory which provides a graded module structure of $K^*(X, \delta)$ over $K^*(X)$;

Other work

- Developed a product on higher twisted K-theory which provides a graded module structure of $K^*(X,\delta)$ over $K^*(X)$;
- Formulated higher twisted K-theory using Fredholm operators;

Other work

- Developed a product on higher twisted K-theory which provides a graded module structure of $K^*(X, \delta)$ over $K^*(X)$;
- Formulated higher twisted K-theory using Fredholm operators;
- Constructed explicit geometric representatives for higher twists given by cohomology classes in special cases:

Other work

- Developed a product on higher twisted K-theory which provides a graded module structure of $K^*(X, \delta)$ over $K^*(X)$;
- Formulated higher twisted K-theory using Fredholm operators;
- Constructed explicit geometric representatives for higher twists given by cohomology classes in special cases:
  - For spheres via the clutching construction;

Other work

- Developed a product on higher twisted K-theory which provides a graded module structure of $K^*(X, \delta)$ over $K^*(X)$;
- Formulated higher twisted K-theory using Fredholm operators;
- Constructed explicit geometric representatives for higher twists given by cohomology classes in special cases:
  - For spheres via the clutching construction;
  - For 5-twists which can be decomposed into the cup product of a 2-class and a 3-class;

Other work

- Developed a product on higher twisted K-theory which provides a graded module structure of $K^*(X, \delta)$ over $K^*(X)$;
- Formulated higher twisted K-theory using Fredholm operators;
- Constructed explicit geometric representatives for higher twists given by cohomology classes in special cases:
  - For spheres via the clutching construction;
  - For 5-twists which can be decomposed into the cup product of a 2-class and a 3-class;
- Computed the higher twisted K-theory of spheres, including an explicit generator of the non-trivial K^1 group via Fredholm operators;

Other work

- Developed a product on higher twisted K-theory which provides a graded module structure of $K^*(X, \delta)$ over $K^*(X)$;
- Formulated higher twisted K-theory using Fredholm operators;
- Constructed explicit geometric representatives for higher twists given by cohomology classes in special cases:
  - For spheres via the clutching construction;
  - For 5-twists which can be decomposed into the cup product of a 2-class and a 3-class;
- Computed the higher twisted K-theory of spheres, including an explicit generator of the non-trivial K^1 group via Fredholm operators;
- Performed computations in some cases where the cohomology contains torsion, including real projective space;

Other work

- Developed a product on higher twisted K-theory which provides a graded module structure of $K^*(X, \delta)$ over $K^*(X)$;
- Formulated higher twisted K-theory using Fredholm operators;
- Constructed explicit geometric representatives for higher twists given by cohomology classes in special cases:
    - For spheres via the clutching construction;
    - For 5-twists which can be decomposed into the cup product of a 2-class and a 3-class;
- Computed the higher twisted K-theory of spheres, including an explicit generator of the non-trivial K^1 group via Fredholm operators;
- Performed computations in some cases where the cohomology contains torsion, including real projective space;
- Computed the 5-twisted K-theory of SU(2)-bundles over 4-manifolds, which are relevant in the setting of spherical T-duality.

Applications

- Classical twisted K-theory is relevant in the study of D-branes in string theory; in the presence of a B-field, D-brane charges take values in a twisted K-theory group.

Applications

- Classical twisted K-theory is relevant in the study of D-branes in string theory; in the presence of a B-field, D-brane charges take values in a twisted K-theory group.
- Work by Bouwknegt, Evslin and Mathai shows that spherical T-duality induces an isomorphism on higher twisted K-theory. They also explain that higher twisted K-theory corresponds to the set of conserved charges of a class of branes in Type IIB string theory.

Applications

- Classical twisted K-theory is relevant in the study of D-branes in string theory; in the presence of a B-field, D-brane charges take values in a twisted K-theory group.
- Work by Bouwknegt, Evslin and Mathai shows that spherical T-duality induces an isomorphism on higher twisted K-theory. They also explain that higher twisted K-theory corresponds to the set of conserved charges of a class of branes in Type IIB string theory.
- The Cuntz algebra $\mathcal{O}_\infty$ can be replaced by other algebras to obtain a different class of twists, such as the CAR algebra which is relevant in quantum mechanics.

Applications

- Classical twisted K-theory is relevant in the study of D-branes in string theory; in the presence of a B-field, D-brane charges take values in a twisted K-theory group.
- Work by Bouwknegt, Evslin and Mathai shows that spherical T-duality induces an isomorphism on higher twisted K-theory. They also explain that higher twisted K-theory corresponds to the set of conserved charges of a class of branes in Type IIB string theory.
- The Cuntz algebra $\mathcal{O}_\infty$ can be replaced by other algebras to obtain a different class of twists, such as the CAR algebra which is relevant in quantum mechanics.
- A major result by Freed, Hopkins and Teleman is that the Verlinde ring of representations of loop groups is equal to the equivariant twisted K-theory of a compact Lie group. There likely exist generalisations of this result to the higher twisted setting, and this is currently being studied by Pennig and Evans.