INSTITUTE FOR GEOMETRY AND ITS APPLICATIONS | THE UNIVERSITY of ADELAIDE

# IGA/AMSI Workshop on

## Geometry of Supermanifolds

15–19 October, 2012

Lecture series by Ugo Bruzzo

**Abstract:** The first lecture will be devoted to give some necessary background: graded algebra, sheaf theory, cohomology. This will most likely extend to the second lecture. Then I will give an introduction to the different possible approaches to defining supermanifolds. Between the second and third lecture I will develop the basic geometry of supermanifolds, including some theory of super vector bundles.

The fourth lecture will be devoted to super Riemann surfaces. In the fifth lecture I will give an example of an application of supermanifold theory to physics, i.e., how to "superize" the moduli space of instantons on a 4-fold and use it to give a description of the BRST transformations, to compute the "supermeasure" of the moduli space, and the Nekrasov partition function.

### Titles of individual lectures

*Lecture 1:* Background: graded algebras, sheaf theory, cohomology

*Lecture 2:* Categories of supermanifolds

*Lecture 3:* Geometry of supermanifolds

*Lecture 4:* Super Riemann surfaces

*Lecture 5:* Applications to physics

## Talks by invited speakers

1. **Tarje Bargheer**, University of Melbourne

   **Title:** Stretching String Topology

   **Abstract:** String topology describe operations associated to the free mapping spaces $M^{S^n}$. I will introduce these from the personal point of view of my thesis. I will then go on to describe ongoing process with how this point of view leads to various extensions of string topology, describing an up-to homotopy version of the intersection product; as well as providing an entangled version of string topology through a twist with Khovanov homology.

2. **Kowshik Bettadapura**, Australian National University

   **Title:** An Introduction to the Stolz-Teichner Program

   **Abstract:** The Stolz-Teichner program began almost a decade ago with the goal of using the framework of field theories to describe cocycles in certain generalsed cohomology theories, such as TMF. While this goal has not been realised yet, some progress has been made in the low dimensional cases. In extending Atiyah's classical definition of a topological field theory, by adding notions of smoothness and supersymmetry, Stolz and Teichner were able to describe cocycles in de Rham cohomology and K-theory. In this talk I will discuss these notions and use the $(0|1)$-dimensional field theories as a case study for modelling cocycles in de Rham cohomology.

3. **Peter Hochs**, Leibniz University, Hannover

   **Title:** Geometric quantisation in the noncompact setting

   **Abstract:** Traditionally, the geometric quantisation of an action by a compact Lie group on a compact symplectic manifold is defined as the equivariant index of a certain Dirac operator. This index is a well-defined formal difference of finite-dimensional representations, since the Dirac operator is elliptic and the manifold and the group in question are compact. From a mathematical and physical point of view however, it is very desirable to extend geometric quantisation to noncompact groups and manifolds. Defining a suitable index is much harder in the noncompact setting, but several interesting results in this direction have been obtained.

   I will review the difficulties connected to noncompact geometric quantisation, and some of the solutions that have been proposed so far, mainly in connection to the "quantisation commutes with reduction" principle. More generally, I will go into some approaches to index theory on noncompact spaces.

4. **David Ridout**, Australian National University

   **Title:** Wess-Zumino-Witten Models on Lie Supergroups

   **Abstract:** The Wess-Zumino-Witten (WZW) models describe bosonic strings propagating on the manifold underlying a (suitably nice) Lie group $G$. Mathematically, these theories are described by the vertex operator algebra built from the untwisted affine Kac-Moody algebra associated to $G$ and have provided a plethora of structures and results for mathematicians to investigate. Physicists are, of course, more interested in supersymmetric strings, so they are naturally led to WZW models on Lie supergroups. These, and their homogeneous space generalisations, are important, for example, in the celebrated AdS/CFT correspondence.

   This talk will review the simplest WZW models, those with $G = GL(1)$ and $G = U(1)$, before discussing recent progress in understanding the simplest super-examples: $G = PSL(1|1)$ and $G = GL(1|1)$. Algebraically, one has a fairly complete picture based on thorough analyses of the untwisted affine Kac-Moody superalgebras. In particular, one has the spectra of the quantum fields, a Verlinde formula, and the appearance of reducible but indecomposable representations. However, a geometric understanding of this picture is still quite vague despite much evidence suggesting that such an understanding should be possible. The aim is to review some of this evidence and its context.