INSTITUTE FOR GEOMETRY AND ITS APPLICATIONS

THE UNIVERSITY of ADELAIDE

IGA Workshop on

Current Groups and Twisted K-theory

2–5 April 2013

Lecture series by Jouko Mickelsson

Abstract: We start looking at how gauge anomalies in quantum field theory give rise to abelian extensions of current algebras. In the hamiltonian formulation of the quantized Dirac field in Yang-Mills background the chiral anomaly defines a gerbe on the moduli space of gauge connections, the Dixmier-Douady class of the gerbe can be computed from families index theorem.

Using the gerbe as input one can define twisted K-theory on the moduli space. In particular, in 1+1 space-time dimensions one can use the supersymmetric Wess-Zumino-Witten model for constructing twisted K-theory classes on compact Lie groups (related to the Freed-Hopkins-Teleman correspondence between twisted K-theory and the Verlinde ring). We shall describe analogous constructions in the case of decomposable Dixmier-Douady classes.

Finally, we shall explain a recent work of F. Wagemann and C. Wockel on group cohomology, relating it to gauge group extensions and transgression in Lie algebra cohomology. More precisely, we shall extend the higher group cocycles to the transformation groupoid setting (motivated by QFT) and discuss potential obstructions in the construction due to a nonvanishing of low dimensional homology groups of the gauge group. The resolution of the obstruction is obtained by an application of the Cheeger-Simons differential characters.

Titles of individual lectures

Lecture 1: Anomalies in quantum field theory

Lecture 2: Current group extensions and gerbes

Lecture 3: Gerbal representations and 3-cocycles

Lecture 4: Twisted K-theory constructions

Lecture 5: Gauge groupoid cocycles and Cheeger-Simons differential characters

Talks by invited speakers

- **Peter Bouwknegt**, Australian National University

  Title: K-theory in Condensed Matter Physics

  Abstract: Topological insulators and superconductors are many-fermion systems possessing an unusual band structure that leads to a bulk band gap as well as topologically protected gapless extended surface modes. It was recently realised that deformation classes of gapped Hamiltonians are naturally classified by K-theory. This classification parallels the classification of the 2 complex and 8 real symmetry classes of Hamiltonians (the '10-fold way') of Altland and Zirnbauer, and naturally leads to a periodic table of topological insulators. In this talk I will give a brief overview of these developments, as well as discuss some open problem.

- **Siye Wu**, University of Hong Kong

  Title: Index gerbe and differential K-theory

  Abstract: We discuss results on determinant line bundles of Bismut-Freed, Quillen and Witten, the notion of index gerbe due to Carey-Mickelsson-Murray and Lott, and the recent index theorem of Freed- Lott in differential K-theory. Then we show, by calculations in differential K-theory, that for a family of Riemannian manifolds of odd dimensions, the index in the odd differential K-group maps to the Deligne cohomology class of the index gerbe. This is a joint work with V. Mathai.