Institute for Geometry and its Applications
School of Mathematical Sciences
University of Adelaide

Workshop on New Currents in Geometry in Australia

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ABSTRACTS OF TALKS

David Baraglia, Oxford University

Coassociative submanifolds: deformations and fibrations

We study the deformation theory of compact coassociative submanifolds of a G2 manifold. This leads us into the study of coassociative fibrations which can be thought of as the G2 equivalent of a special Lagrangian fibration. It has even been conjectured that an analogue of the Strominger-Yau-Zaslow conjecture should hold for these fibrations. We consider a special class of coassociative fibrations, which we call semi-flat, where the fibres are orbits of a torus action of isomorphisms. We show that semi-flat fibrations are locally equivalent to finding minimal submanifolds in a vector space with signature (3,3) metric.

James Borger, Australian National University

Algebraic geometry to arithmetic geometry to absolute geometry

Complex algebraic geometry is ultimately the study of polynomials with complex coefficients. Arithmetic algebraic geometry is then the study of polynomials whose coefficients lie in rings of arithmetic interest, such as the ring of rational numbers. This yields rich arithmetic structures living on top of the plain geometric structure: the more fundamental the ring, the richer the structure. From the usual point of view, this process stops at the ring of integers, the most fundamental ring of all. In this talk, I'll explain a way of digging even deeper and why one might care.

Emma Carberry, University of Sydney

Bubble, bubble, toil and trouble: constant mean curvature surfaces and quaternionic holomorphic geometry

I will describe a number of results regarding the classification of constant mean curvature surfaces, and explain how a quaternionic holomorphic approach to surface geometry is useful and this and other contexts.

Julie Clutterbuck, Australian National University

Capillary surfaces for a compressible liquid

The classical model for a capillary surface assumes that the liquid is incompressible. I will talk about some recent work with Maria Athanassenas in which we relax this assumption. We use techniques of geometric measure theory to show existence of a minimizer of the new energy, and PDE techniques to show that the minimizer satisfies a prescribed curvature equation.

Ilya Kossovskiy, Australian National University

Classification of homogeneous CR-manifolds in dimension 4

E. Cartan classified, up to local CR-equivalence, all locally homogeneous CR-manifolds in dimension 3. We classify, up to local CR-equivalence, all locally homogeneous CR-manifolds in dimension 4. In particular, we classify all non-degenerate locally homogeneous 4-dimensional CR-manifolds in $\mathbb{C}^3$ with non-trivial isotropy group. Using the classification, we present some non-trivial examples of CR-flat homogeneous 4-dimensional surfaces in $\mathbb{C}^3$.

Thomas Leistner, University of Adelaide

Special holonomy in Lorentzian geometry

The holonomy group of a semi-Riemannian manifold is given by all parallel transports along loops based at the same point. It is a useful tool in describing the geometry of the manifold and in classifying parallel objects. The notion of special holonomy refers to semi-Riemannian manifolds that do not decompose as a geometric product, but for which, however, the holonomy group reduces from the full orthogonal group. For Riemannian manifolds special holonomy groups act irreducibly and were classified by Marcel Berger. For Lorentzian manifolds there is no irreducible special holonomy but non-irreducible ones. In the talk I will present the classification of holonomy groups of Lorentzian manifolds in analogy to Berger's list. I will explain the related geometric structures and give some applications for the existence of parallel spinors and for the holonomy of Lorentzian Einstein manifolds.

Pengzi Miao, Monash University

On critical metrics on compact manifolds with boundary

It is known that, on a closed manifold, Einstein metrics with negative scalar curvature correspond to critical points of the usual volume functional constrained to the space of metrics with constant negative scalar curvature. In this talk, we will show how this variational characterization of Einstein metrics can be generalized to compact manifolds with boundary. Precisely, we will describe the critical point equation for the volume functional on the space of constant scalar curvature metrics with a prescribed boundary metric. We will focus on geometric properties of general solutions to the equation and we will also classify all conformally flat solutions. This is a joint work with Prof. Luen-Fai Tam at the Chinese University of Hong Kong.

Peter Milley, University of Melbourne

Mom-technology and small volume hyperbolic 3-manifolds

Using special cellular complexes embedded inside hyperbolic manifolds, I and my colleagues David Gabai and Robert Meyerhoff successfully proved that the Weeks manifold was the minimum-volume compact hyperbolic 3-manifold, proving a conjecture that had been open for over twenty years. In this talk I'll give an overview of Mom-technology and describe connections to other problems such as finding the maximal number of exceptional Dehn surgeries, and finding new ways of constructing hyperbolic structures on 3-manifolds.

Todd Oliynyk, Monash University

The Newtonian limit of general relativity

Einstein's general relativity is presently the most accurate theory of gravity. To completely determine the gravitational field, the Einstein field equations must be solved. These equations are extremely complex and outside of a small set of idealized situations they are impossible to solve directly. However, to make physical predictions or understand physical phenomena, it is often enough to find approximate solutions that are governed by a simpler set of equations. The prime example of this is Newtonian gravity which approximates general relativity very well in regimes where the typical velocity of the gravitating matter is small compared to the speed of light. Indeed, Newtonian gravity successfully explains much of the behaviour of our solar system and is much less difficult to solve. Furthermore, by generalizing Newtonian gravity to the cosmological setting, it appears that Newtonian theory can accurately describe gravity on all scales except in regions of very high curvature such as near compact neutron stars or black holes. Therefore, it is expected that in certain regimes Newtonian gravitational solutions can be used as a starting point for approximating general relativistic ones. This expectation has spawned a large body of research that goes by the name of the Newtonian limit. Although many researchers over many years have investigated this problem, the precise relationship between solutions to Newtonian gravity and solutions to general relativity remained poorly understood until quite recently.

In this talk, I will introduce the main ideas and geometry behind the Newtonian limit. I will also discuss the mathematics issues that arise in trying to rigorously understand the Newtonian limit, and what mathematical results are presently available. Finally, if time permits, I will discuss the post-Newtonian expansion which is an iterative method to produces corrections to Newtonian gravity that take into account relativistic effects such as gravitational lensing, and the loss of energy due to gravitational radiation.

Lawrence Reeves, University of Melbourne

Genus for finitely presentable groups: compact 3-manifolds and link complements

We associate to each finite presentation of a group G a compact CW-complex that is a 3-manifold in the complement of a point, and whose fundamental group is isomorphic to G. We use this complex to define a notion of genus for G and give examples, and also define a notion of a 'closed group'.

Adam Rennie, Australian National University

Geometry of singular spaces

I will give a brief (!) overview of the philosophy of noncommutative geometry as it applies to singular spaces.

Peter van der Kamp, La Trobe University

From integrable lattice equations to integrable flows

For partial difference equations defined on a square, initial values can be given on staircases. By taking periodic initial conditions the equation reduces to a system of ordinary difference equations, or a mapping/correspondence. For integrable equations, integrals for these mappings/correspondences are obtained by taking the trace of a so-called monodromy matrix. Thus, one might be able to establish complete integrability, in the sense of Liouville-Arnold.

I will generalize the construction of initial value problems for more general equations. I will present combinatorial objects in which integrals of reductions of lattice equations are conveniently expressed. And I will show that the flows of the associated Hamiltonian vector fields can be linearized.

Bryan Wang, Australian National University

Geometric cycles and twisted index theory

In this talk, I will explain three equivalent definitions of twisted K-homology and their roles in twisted index theory. Then I report on some of the latest developments, including a geometric model for the D-branes in string theory and twisted string structures in elliptic cohomology.

Craig Westerland, University of Melbourne

The stable topology of Hurwitz spaces

Hurwitz spaces are moduli spaces of branched covers of Riemann surfaces. We study the behaviour of the topology of these spaces as the number of branch points grows. In certain circumstances, we prove that the homology of Hurwitz spaces of covers of the Riemann sphere stabilises with the number of branch points. Furthermore, we identify the stable homology of these spaces in terms of the classical homotopy theory of iterated loop spaces. Time permitting, we will discuss applications of these results to certain conjectures in arithmetic geometry, such as function field analogues of the Cohen-Lenstra heuristics. This is joint work with Jordan Ellenberg and Akshay Venkatesh.

Feng Xu, Australian National University

Laplacian flow for G_2-structures

A G_2-structure on a 7-dimensional manifold M is given by a definite 3-form σ. Such a G_2-structure is called closed if σ is closed, and torsion-free if σ is both closed and co-closed. The Riemannian holonomy of a torsion-free G_2-structure is contained in the Lie group G_2. In particular, the underlying metric will be Ricci-flat. The Laplacian flow deforms the G_2-structure along its Hodge Laplacian. In this talk, I will review G_2-structures and G_2-holonomy, and introduce Laplacian flow. After this, I will present joint work with R. Bryant on short-time existence of the Laplacian flow in the category of smooth closed G_2-structures. Then I will talk about joint work with Rugang Ye on dynamic stability of torsion-free G_2-structures under the Laplacian flow.

Hao Yin, University of Queensland

On stability of the hyperbolic space form under the normalized Ricci flow

We study the normalized Ricci flow from a slight perturbation of the hyperbolic metric on $\mathbb{H}^n$. It is proved that if the perturbation is small and decays sufficiently fast at infinity, then the flow will converge exponentially fast to the hyperbolic metric when the dimension $n > 5$. This is joint work with Haozhao Li (East China Normal University, China).